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TRENDOVI U POSLOVANJU
NAUČNO-STRUČNI ČASOPIS

VISUALIZATION OF SOLVING PROBLEMS WITH SEQUENCES OF NUMBERS: STATISTICAL COMPARISON OF THE TWO METHODS

VIZUELIZACIJA REŠAVANJA ZADATAKA SA NIZOVIMA BROJEVA: STATISTIČKO POREĐENJE DVE METODE

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Sažetak

Dvadeset prvi vek je doneo narastajuću upotrebu informaciono-komunikacionih tehnologija u mnogobrojnim sferama ljudskog života. Takođe i u obrazovanju. U procesu učenja, učenici se sve više usmeravaju na korišćenje savremenih tehnologija i programskih paketa. Ispitivanjem kvaliteta nastave matematike u osnovnoj školi konstatovali smo naučenost učenika da koriste savremene tehnologije ali i nedostatak vizuelno-logičkog pristupa u rešavanju matematičkih zadataka. U ovom radu pokazujemo kako uvođenje figurativnih brojeva u nastavu matematike i odabranih primera koji demonstriraju uočavanje zakonitosti među brojevima, može uputiti učenike u vizuelno-logički pristup rešavanju zadataka sa skupovima brojeva. Organizovali smo tročasovno vežbanje u eksperimentalnoj i kontrolnoj grupi tokom kojeg je eksperimentalna grupa radila sa figurativnim brojevima a kontrolna sa odabranim primerima. Na pre-testu obe grupe su pokazale nizak stepen pripremljenosti za uočavanje zakonitosti među brojevima i među njima nije bilo statistički značajne razlike. Na post-testu obe grupe su pokazale napredovanje u odnosu na pre-test pri čemu je eksperimentalna grupa postigla veće napredovanje. Istraživanje je pokazalo da rad sa figurativnim brojevima i odabranim primerima za demonstriranje uočavanja zakonitosti među brojevima doprinosi razvoju učeničkih sposobnosti u uočavanju zakonitosti među brojevima i primeni uočene zakonitosti u rešavanju zadataka.

Abstract

The twenty-first century has brought the growing use of information and communication technologies in many spheres of human life. Also in education. In the learning process, students are increasingly focused on the use of modern technologies and software packages. Exploring the quality of maths teaching in primary school, we found students' knowledge of using modern technologies but also the lack of visual-logical approach in solving mathematical problems. In this paper, we show how the establishment of figurative numbers and selected examples that demonstrate the observation of legality, can direct students to a visual-logical approach to solving tasks with sets of numbers. We organized a three-hour exercise in experimental and control group, during which the experimental group was working with figurative numbers and the control group with selected examples. On the pre-test both groups showed a low degree of ability to observe legality among the numbers. On the post-test, both groups achieved progress compared to the pre-test, although the experimental group achieved greater progress. The research has shown that dealing with figurative numbers and selected examples that demonstrate the observation of legality, contributes to the development of pupils' abilities to perceive the laws of numbers and to apply the laws observed to solving problems with sets and arrays of numbers

Ključne reči: vizuelno-logički pristup, uočavanje zakonitosti, figurativni brojevi, Studentov t-test

Keywords: visual-logical approach, perception of laws, figurative numbers, Student's t-test

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1. Introduction

After spending many years of professional work in maths teaching, we realised that the visual-logical approach in solving mathematical problems is represented very little in the teaching of mathematics. Primary and secondary school pupils are primarily trained in mastering the formulas and algebraic procedures by which they come to the solution of the given task. It is necessary for the student to form an equation (or equations) with one or more unknowns by connecting the given data in the task and then to solve the task using the acquired knowledge. Therefore, a very small number of students, or of teachers of mathematics, will reach for the visual-logical way of solving the problem. It is noticeable that students automatically think about equations or the system of equations that will "help them solve their assigned task".

Elementary school students at the very beginning of their mathematics education are directed towards the obvious relations between the objects. During the first and second grades of elementary school, obvious teaching is noticeable but over time it turns into directing students to master the procedures and formulas and their application in solving tasks. Later on in their schooling, students are directed towards applying a visual-logical approach in mathematics learning and solving mathematical tasks only on very rare occasions. In the learning process as well as in solving mathematical problems, visualisation and representation are of the greatest importance [2][12].

The authors of this paper have conducted several researches with primary and secondary school students. The research conducted with first-grade high school students was described in our paper [28]. This research has shown that working with figurative numbers is useful for students in multiple ways. It contributes to developing a student's ability to spot the rules pertaining to these numbers and to solve the problems with numerous sets more successfully. Additionally, spotting the laws between numbers contributes to the longer-term memory of numerous data among which a certain legality exists.

In this paper we are showing the results of the research conducted with eighth grade students of primary school. Our research began by checking the ability of students of the eighth grade to solve various tasks with natural numbers by observing the laws. When we asked students how much the sum of the first 1000 natural numbers is, we received only a small number of correct answers. An even smaller number of correct answers was obtained for the sum of odd numbers smaller than 100. Neither was the simple task of noticing the legality among given numbers and typing the missing number into an empty field done better. We decided to introduce students to selected examples that demonstrate the observation of legality among numbers. In order to compare the results at the end of the research we formed two groups of students, experimental and control. Students of the experimental group were familiar with the polygonal figurative numbers and the laws among them, while students of the control group were familiar with other examples that demonstrate legality among the numbers. There were 4 departments in both groups selected, so that the average grade in mathematics is approximately the same as the number of students in groups. In all departments, we also organized collaborative groups composed of 3 members, because cooperative learning is recommended by many researchers as an extremely effective learning approach [10][7]. We also wanted to help the development of the constructive thinking of students through the planned lesson. One part of the planned teaching we realised by computer and the programme package GeoGebra. Instructing students to use modern technologies, software packages and computers actively is of great importance in modern education [11].

After 3 hours of work with selected examples based on the observation of legality among the numbers, the students of both groups were tested. The results of both the experimental and control groups at the pre-test were approximately equal. In the post-test, the results of both groups were better than on the pre-test. However, the experimental group achieved better results than the control group. We used Student's t-test in the statistical analysis.

Our work consists of 6 sections. After the introduction, the reader will get acquainted with the presence of figurative numbers in mathematics. After that, we present the contemporary educational processes applied in our work, the methodology of our research, the obtained experimental results and the conclusions we came up with after the research.

2. Figurative numbers

The beginners of figurative numbers were Pythagoras and the Pythagoreans. They were the first to start displaying numbers with the figures of triangles, squares and rectangles. Over time, the triangular and the square numbers were complemented with the pentagonal, hexagonal and a whole class of m-angular polygonal numbers, including the pyramidal, the polyhedral and other figurative numbers.

Figurative numbers can be represented by a discrete geometric pattern with equally spaced points, with each point representing a unit (Figure 1). They can also be represented by an algebraic formula by which all numbers of this type are generated. For triangular, square, pentagonal and hexagonal numbers, the corresponding algebraic formulas are: $n\cdot(n+1)/2$; n^2; $n\cdot(3n-1)/2$; $n\cdot(2n-1)$, respectively. Taking the values of the natural numbers for ***n***, in order, we obtain a series of triangular, square, pentagonal and hexagonal numbers.

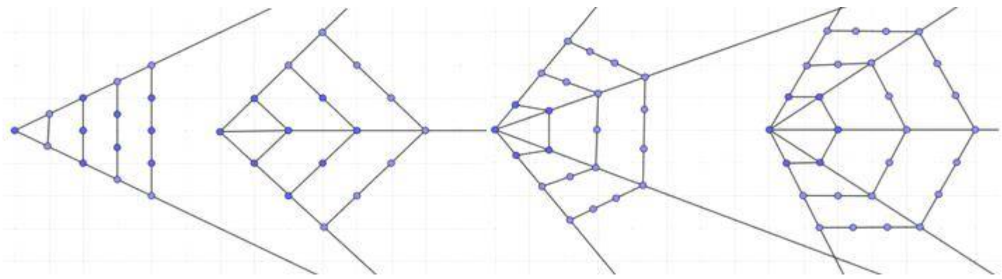


Figure 1: Triangular, square, pentagonal and hexagonal numbers

Geometric interpretation of figurative numbers allows visual observation of their properties and thus makes them suitable objects for mathematical learning. That is why we are not surprised with their multicurrent presence in mathematics. From the Ancient Period to the Contemporary Age, many well-known mathematicians have dealt with them: Pythagoras of Samos, Nicomachus of Gerasa, Theon of Smyrna, Diophantus of Alexandria, Leonardo Pizano Fibonacci, René Descartes, Pierre de Fermat, Blaise Pascal, Leonhard Euler, Karl Friedrich Gauss, et al. [8].

Figurative numbers are a subject of interest for many scientists in the modern age. Matila Ghyka, when dealing with philosophy and number mysticism, starts his researches with Pythagorean figurative numbers as the basis of number theory [14]. Michael Bennett deals with the definition of the ***n***-th tetrahedral number, which is the sum of m square numbers [4]. Interesting results were obtained by displaying natural numbers by the collection of triangular and squares numbers [29]. Also by presenting integers numbers as the sums of triangular numbers [30] and in the solving of Diophantine equations [6][16].

The figurative numbers are associated with Pythagorean triples of numbers, Pascal's triangle, perfect numbers, Fibonacci's series of numbers and other numbers used in mathematics [8]. They are present in many mathematical fields. The results of our research with first-grade high school students [28] confirm that figurative numbers are a good choice for demonstrating and developing the visual-logical approach in solving problems with numerous data.

3. Contemporary educational processes

3.1. New technologies in teaching mathematics

The twenty-first century as the era of widespread information and communication technologies requires technologically educated people, capable of actively using the new technologies. One of the main tasks and objectives of modern education is to introduce pupils to new technologies and provide training for their active use. This goal will be easier to achieve if new technologies are used in the realisation of teaching [25]. New technological advancements have significantly increased the range of teaching resources in education and also set up new demands for teachers [32][5]. Facing new challenges, contemporary teachers need to be prepared to use new teaching technologies for more effective teaching [21][34]. The assumption of an active role by all participants in the education process is a prerequisite for successful teaching and learning.

The development of hardware has contributed to the development of software packages as well, such as GeoGebra, Cabri Geometry, Geometer's Sketchpad, etc. Many researchers have demonstrated the effectiveness of mathematical learning when maths software packages are applied [3][17]. Also, testing the performance of GeoGebra and visualization in mathematics teaching in the IWB (Interactive White Board) equipped classrooms has been well presented in the papers of researchers [24][37].

Many researches have shown that the integration of information and communication technologies into teaching processes contributes to the development of active thinking among students by stimulating their creative thinking [1][36].

3.2. Paradigms and constructivism

Paradigms occupy a significant place in the process of learning and solving tasks. They contribute to faster and easier problem-resolving by switching to an isomorphic problem whose solution is known. The solution of the great mathematician Gauss, who as a 10-year-old boy noticed laws by which he quickly calculated the sum of the first 100 natural numbers, is an excellent paradigm for calculating the sums of various series of numbers that have a similar legality. According to Gestalt theory, in solving a problem, the key role is played by the ability to perceive and understand a problematic situation that can be solved by understanding the relationships between elements of a problem situation [22]. Figurative numbers with their image presentation and the laws that apply to them are very interesting and easy to understand for students, and they can be a very good tool for presenting paradigms and developing constructive thinking [27].

Constructivism is an access to education and learning based on the assumption that the student is creating an opinion about new knowledge and information connecting them with the already existing knowledge. Even though the construction of knowledge is an individual process, the students realise it by interacting with other people. In that context, the work of students in cooperative groups, the exchange of opinions, discussions and explanations are of major importance. The work of teachers during the class is also of the great importance. Constructivism improves students' intellectual development, substitutes for memorisation in learning, and is an excellent alternative to traditional methods in education [19]. The advantages of constructivism in teaching practice are also pointed out by other authors [13][26]. What is more, learning through solving problems as part of the education system is accepted as an effective paradigm of learning [38].

In this paper, we present the introduction of eighth-grade students to examples which improve constructive perception and can be paradigms for many problems which students face during their education. The lesson is realised with computer support and with work in collaborative groups. Nowadays, this approach is considered one of the most advanced in improving learning and teaching [15].

4. Research methodology

4.1. Tasks and goal of the research

The research tasks in this paper to obtain the answers to the following research questions:

- Are the eighth grade primary school students trained to notice the legality among the numbers when solving problems with numerous sets and numerous arrays?
- Does the usage of figurative numbers and selected examples in solving maths problems contribute to developing the ability to spot the legality among the numbers and will students then be more capable of solving the tasks with series of numbers.

The aim of the research in this paper is examining and determining the contribution of figurative numbers and selected examples to the development of abilities in noticing legality between elements of sets of numbers, among the eighth grade primary school students.

4.2. Research hypotheses

The hypotheses in this study are as follows:

- Less than 25% of students in the eighth grade of elementary school will be able to solve the tasks where spotting the legality among the numbers is necessary.
- Working with figurative numbers and selected examples will help train students to solve tasks by applying the laws observed.

4.3. Research methods and instruments

The research was conducted during October 2019 at the "Rade Drainac" Elementary School in Belgrade. The authors of this paper carried out the research together with primary school teachers. There were 8 departments in the eighth grade with a total of 235 students. At the beginning of the research, we carried out testing of students in order to examine their learning ability as regards solving various tasks with natural numbers by observing laws. The pre-test set the following tasks:

- What is the sum of the first 1000 natural numbers?
- What is the sum of the odd numbers below 100?
- Taking into account the laws valid for the first two puzzle numbers, fill in the empty space in the third puzzle (Figure 2):

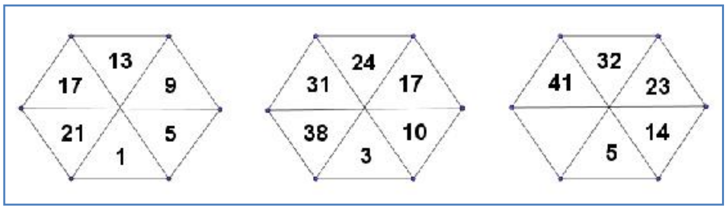


Figure 2: The third task on the pre-test

The pre-test was intended to show the students ability to notice the legality among the numbers and to apply them to solve the given tasks.

The pre-test results are shown in Section 5 (Table 1).

Only 37 of a total of 235 students in the eighth grade solved the first task - which is to say, 15.7%; 31 pupils, or 13.2%, solved the second task; the third task, 85 students or 36.2%. The total number of solved tasks is 153 or 21.7%.

The results obtained at the pre-test can be characterized as evidence for our first hypothesis that less than 25% of students would be able to solve the problems where spotting the legality among numbers is necessary.

We formed an experimental and a control group of students, after consideration of their average grades in mathematics and the number of students in the departments. The experimental group consisted of classes 8_1, 8_3, 8_5 and 8_7 (a total of 117 students), and the control group consisted of classes 8_2, 8_4, 8_6 and 8_8 (a total of 118 students). The average of grades in mathematics as well as the number of students in the groups was about the same. In the departments of the experimental and control groups, small three-member groups were formed for collaborative learning. For these groups, we chose the pupils with different levels of mathematical knowledge. Collaborative group learning is more effective than self-study because members of the group help each other during the work. Such groups are recommended by numerous researchers of effective learning [20][31]. During collaborative learning, students who do not understand the matter ask questions, the students who do understand help them and thus, in explaining to them, determine their own knowledge. This work brings mutual benefits. Collaborative learning develops teamwork and responsibility in the workplace because each individual is responsible not only for his/her own learning, but also for the learning of the other members in the group [23]. In order to avoid negative feelings and bad associate relationships in the group, students are allowed to group themselves independently, but respecting the principle of different levels of mathematical knowledge. A good collaboration is a prerequisite for effective collaborative learning [9].

For both groups of students, we prepared a three-hour introduction to the selected examples. The aim was to demonstrate the spotting of the connection between the elements of the problem situation and the way of solving it by using the observed link. For the pictorial display of figurative numbers in the experimental group and the chosen examples in the control group, we chose the GeoGebra programme package because of its simple use and the fact that the students were already introduced to the work in GeoGebra. Presenting figurative numbers with the help of dots, in which every dot presents a unity is very easily achievable with the GeoGebra graphic display. The drawing of closed polygonal lines, and other geometrical objects used during the study with the control group, and their colouring, are also easily achievable. The dynamicity of GeoGebra was not used, nor its other possibilities.

4.4. Working with an experimental group and a control group

At the beginning of the first hour, students in the experimental group were introduced to the triangular numbers shown in Figure 3.

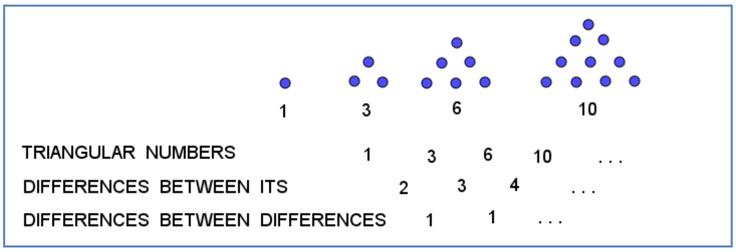


Figure 3: Display of triangular numbers

Then, the students were required to determine the next number in a series of triangular numbers and to portray it in GeoGebra. The groups worked together on the assigned task. Right from the start, the next triangular number was determined. Several students recognized that the task should be solved by applying differences. They concluded that the next number in the series of differences must be 5, so by adding number 5 to number 10 they obtained the number 15 as the next triangular number. Applying the observed image law, several groups drew 5 more points and concluded that the next number in the series of triangular numbers would be number 15. However, a large number of groups failed to reach a solution.

Groups that correctly solved the task helped other groups arrive at the correct solution. Then all the groups worked on the GeoGebra image number 15.

After that, students were acquainted with the square, pentagonal and hexagonal numbers analogously, and they also determined the next number in a series and displayed it in GeoGebra. During the work, teachers encouraged students to link the new tasks with a task known to them, to ask questions and discuss the solution obtained.

At the beginning of the second hour, the students were assigned a second task, in which they were asked to write down 3 more elements in a series of differences and another 3 elements in a series of triangular, square, pentagonal and hexagonal numbers. In this way, students were instructed which way to think. They used the following facts: first, that it is necessary to determine the next difference between the two figurative numbers, then add it to the previous figurative number and thus form the following figurative number.

After that, we asked the students to determine the 30th square number. All groups specified the required number, but in the following way: $30^2 = 900$. Afterwards we asked them to determine the 30th triangular number. The students' comments were something along the lines of:

"How are we going to count it? We do not have a formula."

We then showed the students a procedure for determining the 30th triangular number.

At the beginning of the third hour, the students were required to determine the 21th triangular number. During their work, teachers talked with the group members and recorded their observations and answers. In all groups, the students repeated the procedure from the previous task, noting that the 21st triangular number was equal to the sum

$$1 + 2 + 3 + . . . + 21.$$

However, not even one group accurately calculated that sum. Grouping by two numbers, they received their sum of 22. Concluding that there were 10 such pairs they then obtained the next solution: $10 \cdot 22 = 220$. In the other groups, the opinions were that these pairs had 11, so they received a solution of $11 \cdot 22 = 242$. Since neither of the solutions were correct, we gave the students instructions how to solve this task. Then, students were asked to calculate the 12th pentagonal number. Guided by the example of determining the 30th triangular number and using the differences between two adjacent collections, the students were able to determine the twelfth pentagonal number.

After completing this task, students from the groups who had found the exact solution were invited to explain how they achieved this to the other students, who had not obtained the correct result.

Then the task was solved with the determination of the 9th hexagonal number. It was noted that the students were solving this task much faster and easier than the previous task.

After this task, work with the experimental group was completed. The procedures and results of the students' work presented show that students of the eighth grade of elementary school are able, after only a few examples based on the observation of the rules applying to figurative numbers, to notice various laws among those numbers. They were also able to apply them in order to resolve the task.

Students of the control group at the beginning of the first hour were introduced to the Pythagorean procedure for determining the collection of odd numbers (Figure 4_A).

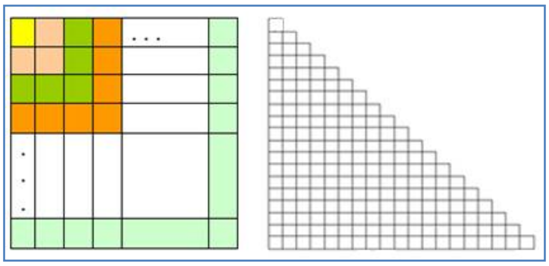
Then students were required to determine the sum of the first 6 odd numbers and to display it in GeoGebra, as in the previous example. After that, they were required to calculate the sum of the following numbers:

$1 + 3 + 5 + 7 + . . . + 29.$

Then the students were assigned the following task:

To determine the total number of square plates required for the paving on one side of a staircase with 19 steps.

With the text of the task, students received an image of a staircase on the computer (Figure 4_B). After the completion of the time scheduled for solving this task, the students were shown the solution with the addition of a staircase in an inverted position, which gave a rectangle in which the total number of square plates was 19 · 20, on the basis of which it was concluded that the required number of square plates was 190.



4_A 4_B

Figure 4: View of collection of odd numbers and the staircase

At the beginning of the second hour, the students of the control group were introduced to the Gaussian procedure of calculating the collection of the first 100 natural numbers. After that, students were required to calculate the following sums:

- 1 + 2 + 3 + . . . + 10 + 11 + . . . + 20
- 1 + 2 + 3 + ... + 22 + 23 + 24 + . . . + 44 + 45
- A collection of all even numbers smaller than 100.

All tasks were resolved on time in all groups, with the help of the teachers, or with the help of the groups who had reached the solution independently.

During the third hour, the students solved the following tasks:

1. Observe the rules for forming a given numerical series and write 3 more numbers:

- 2 , 5 , 8 , 11 , 14 , 17 . . .
- 1 , 2 , 4 , 7 , 11 , 16 , 22 , 29 . . .

2. Regarding the laws applied in the first three puzzle numbers, fill the empty space in the fourth puzzle. (Several examples of number puzzles are given.)

During this time, teachers asked the students to analyse the set tasks, to connect them with the tasks whose solutions were known. They helped the students come to the solution, and the students who quickly understood the process of work helped the other members of their group to progress.

After the completion of the three-hour work, all the students of the eighth grade did the test for checking the acquired ability in noticing the legality among the numbers and applying observed laws in solving the tasks. The results of the test with an analysis of students' performance are enclosed in Section 5.

5. Results of the test

In this section, we present a statistical analysis of the pre-test and post-test results based on students' achievements, expressed by the number of tasks solved and the number of points obtained on tests, respectively. Student's t-test of difference between the means of the two large independent samples was applied in each analysis.

5.1. Pre-test results

The aim of the pre-test was to check the knowledge of the students and their capability at noticing lawfulness among the numbers. On the basis of the results of the pre-test, shown in Table 1, we obtained the answer to our first research question and concluded that students were not being taught about such an approach to analysing and solving tasks.

Table 1: Pre-test results

Department	Number of students	The number and percentage of students who found correct solutions		
		1.task	2.task	3.task
8_1	31	5 (16.1%)	4 (12.9%)	9 (29.0%)
8_2	30	4 (13.3%)	3 (10.0%)	10 (33.3%)
8_3	30	4 (13.3%)	4 (13.3%)	8 (26.7%)
8_4	29	5 (17.2%)	4 (13.8%)	12 (41.4%)
8_5	28	4 (14.3%)	3 (10.7%)	12 (42.8%)
8_6	30	6 (19.8%)	5 (16.5%)	13 (42.9%)
8_7	28	5 (17.8%)	5 (17.8%)	11 (39.3%)
8_8	29	4 (13.8%)	3 (10.3%)	10 (34.5%)

The statistical pre-test results are shown in Table 2.

Table 2: Statistical results of the pre-test

Group	Number of students	Arithmetic mean	Standard deviation	Test of the difference between arithmetic means	
				t-value	p (2-Tailed))
Experimental	117	0.635	0.084873	0.528	0.596
Control	118	0.669	0.096797		

The pre-test also showed that there was no statistically significant difference between the experimental and control groups. On the basis of the average number of students' points and calculated values for the arithmetic mean and standard deviation, a t-value of 0.528 was obtained. The resulting t-value of 0.528 was less than the limit value for a significance threshold of 0.05, indicating that the difference between the experimental and the control groups was not statistically significant at the level of significance of 0.05.

5.2. Post-test result

The post-test includes checking the students' progress after the three-hour practice of the visual-logical approach in solving problems through the observation of laws among the numbers, as we have described in the previous section. To solve the tasks in the post-test, it was necessary to see the laws pertaining to the numbers. Their solutions provided the answer to the second research question. The following tasks were set for the post-test:

- Calculate the sum of the first 200 natural numbers.
- Calculate the sum of all odd natural numbers less than 100.
- Write the following number in the series of numbers: 2, 4, 8, 14, 22, 32, 44 . . . .
- Taking into account the lawfulness in the first three puzzle numbers, fill the empty space in the fourth puzzle (Figure 5):

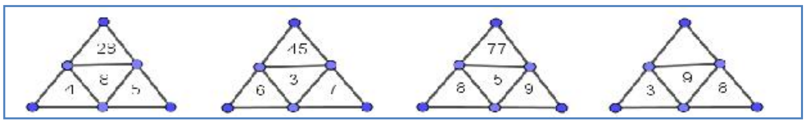


Figure 5: Fourth post-test task

The exact solution in each task received the score of 5 points. The post-test results by classes are shown in Table 3.

Table 3: Post-test results by classes

Depart-ment	Number of students	Number of students who provided correct solutions				Total points in department	Average points per students
		1.task	2.task	3.task	4.task		
8_1	31	21	19	18	22	400	12.90
8_2	30	17	15	13	15	300	10.00
8_3	30	20	20	17	23	400	13.33
8_4	29	18	17	13	16	320	11.04
8_5	28	24	22	21	22	445	15.89
8_6	30	19	18	13	19	345	11.50
8_7	28	25	24	22	21	460	16.43
8_8	29	17	14	12	15	290	10.00

In the first task, the students of the experimental group achieved a total 90 correct answers (out of a possible 117), which was 76.9%, while the control group members achieved 71 correct answers (out of a possible 118), which was 60.2%. Regarding the second task for the experimental group, there were 85 correct answers, or 72.6%, and in the control 64, or 54.2%. In the third task, the difference in the achievements of the experimental and the control groups was even greater. The experimental group provided 78 correct answers, or 66.7%, while the control group produced 51 correct answers, or 43.2%. For the fourth task, the experimental group put up a performance of 75.2% (or 88 correct responses), while the control group arrived at 55.1% (or 65 correct responses). The difference in performance between the experimental and the control groups is best seen in the graphical presentation of the results (Figure 6).

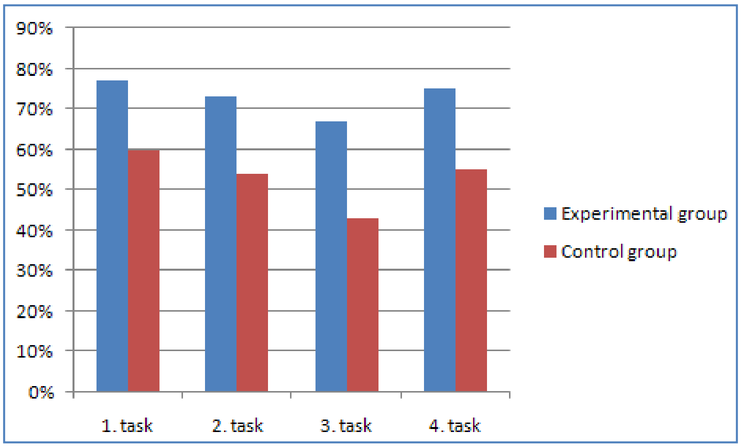
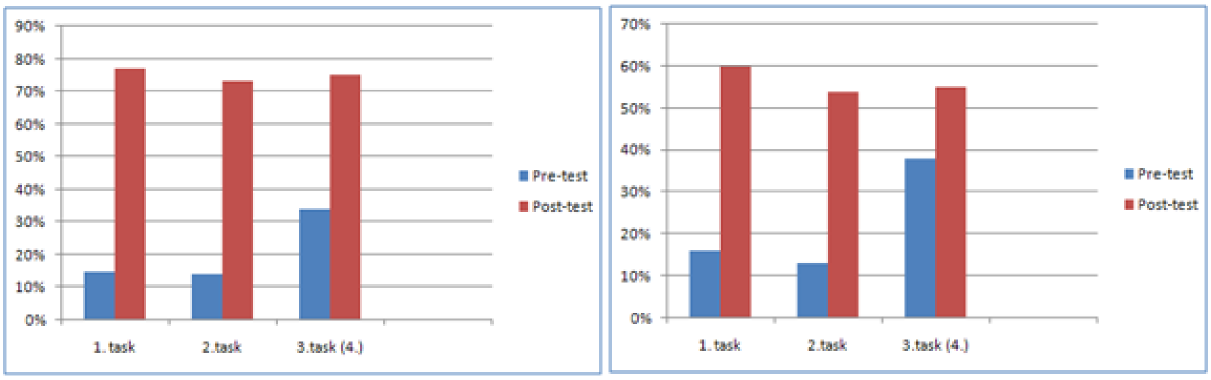


Figure 6: The performance of the experimental and control groups in the post-test

In the post-test, the first and fourth tasks were of the same weight and complexity as the first and third tasks on the pre-test, while the second task was identical in both tests. By comparing the results in these post-test and pre-test tasks, both of the groups made progress in their performance. In the pre-test, the students of the experimental group achieved 18 correct answers (or 15.4%) in the first task, 16 (or 13.7%) in the second, and in the third, 40 (or 34.2%), while the students in the control group achieved 19 (or 16.1%) in the first task, 15 (or 12.7%) in the second, and in the third 45 (or 38.1%). The progress of both groups achieved after the three-hour practice with the visual-logical approach in solving the tasks with numerous data is clearly visible in the graphic presentation of these results (Figures 7_A and 7_B).



7A 7B

Figure 7: Pre-test and post-test results in the experimental group (7A) and the control group (7B)

The chart also indicates that the experimental group achieved greater progress compared to the control group. Regarding the first task, the experimental group achieved 61.5 percent better results compared to the pre-test, and the control group 44.1 percent. Regarding the second task, the experimental group were 58.9 percent better, and the control group 41.5 percent. And regarding the third task, the experimental group achieved for 38 percent more successful results, and the control group, 17 percent.

In the post-test, the greatest difference in performance between the experimental and the control groups was achieved in the third task, where the detection of laws among the elements of a numerous series was required. In this task, the students of the experimental group provided 78 correct answers (or 66.7%), and the students of the control group,51 correct answers (or 43.2%). Also, the results obtained showed that working with figurative numbers contributes to the development of the ability to notice the laws among the numbers and to the more successful solution of tasks using the laws observed. And that is proof of our second hypothesis.After the realisation of the three-hour exercise with the visual-logic approach to solving problems with numerous strings on a different ways, both groups, the experimental and the control, made progress. That fact shows that figurative numbers are not the only instruments which improve the ability of students to observe the laws between the numbers and to apply the rules observed when solving problems, but they are still effective.

The statistical post-test results show that the difference in the knowledge between the students of the experimental and the control groups after the three-hour exercise was statistically significant (Table 4).

Table 4: Statistical post-test results

Group	Number of students	Arithmetic mean	Standard deviation	Test of the difference between arithmetic means	
				t-value	p (2-Tailed))
Experimental	117	14.638	1.5419	4.778	0.000
Control	118	10.635	0.6555		

The resulting t-value of 4.778 is greater than the limit value for a significance threshold of 0.05. This indicates that the difference in the average number of points achieved in the experimental and the control groups at the post-test was statistically significant, with a significance level of 0.05.

Statistical analysis shows that the students' achievements in perceiving the laws of numbers and applying the laws observed to solving problems are better after practicing the visual-logical approach based on figurative numbers.

6. Conclusion

Taking into account the results of the students' achievement both at the pre-test and the post-test, we can conclude that:

- Eighth grade primary school students were not thought to solve problems with numerous data by observing the laws among the numbers;
- Use of figurative numbers and selected examples in solving maths problems contributes to developing the ability to spot laws among numbers. Also, it contributes to more successful solutions to problems in which spotting the laws pertaining to numbers is necessary.

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